DOCUMENT RESUME

ED 275 698 TM 860 575

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TITLE	Assessing the Dimensionality of the GMAT Verbal and Quantitative Measures Using Full Information Factor Analysis.
INSTITUTION	Educational Testing Service, Princeton, N.J.
SPONS AGENCY	Graduate Management Admission Council, Princeton, NJ.
REPORT NO	ETS-RR-86-13
PUB DATE	Mar 86
NOTE	21p.
PUB TYPE	Reports - Research/Technical (143)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Business Administration Education; College Entrance Examinations; *Factor Analysis; *Factor Structure; Graduate Study; Higher Education; *Item Analysis; *Latent Trait Theory; Mathematical Models; Mathematics Tests; *Maximum Likelihood Statistics; Verbal Tests
IDENTIFIERS	*Graduate Management Admission Test; *TESTFACT; Unidimensionality (Tests)

ABSTRACT

When the three-parameter logistic model and item response theory are used to analyze Graduate Management Admission Test (GMAT) data, there are problems with the assumption of unidimensionality. Linear factor analytic models, exploratory factor analysis programs, and the comparison of item parameter estimates for heterogeneous and homogeneous subsets also present difficulties. A new method of assessing the dimensionality of binary data is now available. TESTFACT is a computer program which can be used to perform full information factor analysis, using the marginal maximum likelihood method to estimate reparameterized discrimination and difficulty parameters for multidimensional item response models. The lower asymptote for each item is treated as a known constant whose value is input by the program user. TESTFACT allows a stepwise factor analysis to be performed. First a one-factor solution is performed, then a two-factor solution. The difference between chi squares for the two solutions is used to test whether the added factor is statistically significant. When TESTFACT was applied to both quantitative and verbal GMAT items, a dominant first factor and two additional, considerably weaker, factors were found. (Author/GDC)

RESEARCH REPORT

ASSESSING THE DIMENSIONALITY OF THE GMAT VERBAL AND QUANTITATIVE MEASURES USING FULL INFORMATION FACTOR ANALYSIS

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Educational Testing Service
Princeton, New Jersey
March 1986

Assessing the Dimensionality of the GMAT Verbal and Quantitative Measures Using Full Information Factor Analysis[1]

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Educational Testing Service

This Research was Sponsored by the
Graduate Management Admission Council

March 1986

[1] The consultation and review of Lawrence Hecht, Robert Mislevy, Nancy Petersen, and Martha Stocking is gratefully acknowledged. Also, thanks to Peter Pashley for getting TESTFACT up and running. The opinions expressed herein are those of the author and do not neccesarily reflect those of Educational Testing Service, the Graduate Management Admission Council, nor any of the reviewers and consultants.

INTRODUCTION

Kingston, Leary, and Wightman (1985) explored the applicability of item response theory methods for the Graduate Management Admission Test. In their report they assessed the fit of the three-parameter logistic model to GMAT data in a number of ways. In particular, they assessed the appropriateness of the unidimensionality assumption using two methods: 1) analysis of previous exploratory factor analyses (i.e., Swinton & Powers, 1981) and 2) comparison of item parameters estimated both for homogeneous and heterogeneous subsets of items. These analyses indicated that the Verbal and Quantitative measures of the GMAT each probably has two major dimensions, and possibly a number of minor dimensions.

Each of these types of analyses, however, has a theoretical drawback for assessing dimensionality in an IRT framework. Although the three-parameter logistic model assumes unidimensionality, it does not require that the dimension be linearly related to the scored item responses (right vs. wrong) from which the trait is drawn. Thus, linear factor analytic models might present a misleading picture. Also, commonly available exploratory factor analysis programs do not indicate whether the factors are statistically significant. The comparison of item parameter estimates for items calibrated both in homogeneous and heterogeneous subsets also provided no indication as to whether or not the differences were statistically significant.

A new method of assessing the dimensionality of binary data is now available. TESTFACT (Wilson, Wood, & Gibbons, 1984) is a computer program which can be used to perform full information factor analysis (Bock, Gibbons, & Muraki, 1985). Full information factor analysis, as implemented in TESTFACT, uses the marginal maximum likelihood method (Bock & Aitken, 1981) to estimate (reparameterized) discrimination and difficulty parameters for multidimensional IRT models. The lower asymptote for each item is treated as a known constant whose value is input by the program user. TESTFACT allows a stepwise factor analysis to be performed. First a one-factor solution is performed, then a two-factor solution. The difference between chi squares for the two solutions is used to test whether the added factor is statistically significant. A third, fourth, or more factors can be added, but computation time and expense increases exponentially with the number of factors.

Description of the Test

The data presented in this report were obtained from the Graduate Management Admission Test, which reports Verbal, Quantitative, and Total scaled scores. The test consists of eight separately timed sections, two of which were not used as part of this study. The reported scores are derived from six of the sections. The Verbal measure consists of 85 items administered in three sections: reading comprehension (25 items), sentence correction (25 items), and analysis of situations (35 items). The Quantitative measure includes 65 items: two sections of problem solving items (40 items total) and one section of data sufficency items (25 items). Examples of all item types are available in the GMAT Bulletin of Information (GMAC, 1985).

Data Collection

Data analysis was based on random sample of 5,000 examinees who took GMAT form 3FBS1 at the January 1983 administration.

Analysis

A scored item tape was created for input into the analyses. Although the GMAT is scored operationally as number right minus one-quarter number wrong, for the purpose of this study all items were scored 0 if wrong and 1 if right. For the calculation of tetrachoric correlations there are three options: 1) delete all examinees who omitted any item, 2) do pairwise calculation of tetrachorics, deleting examinees who omitted one of the pair of items, and 3) code omitted items as wrong. In addition, TESTFACT allows all omitted items following the last item to which the examinee responded to be treated as not presented. This was not done as such treatment would make difficult the finding of any speed factor, should one exist.

A one factor, two factor, and three factor full information factor analysis was run separately for the GMAT Verbal and Quantitative measures. For the verbal analysis, the one- and two-factor solutions converged readily, but for the three-factor solution a Bayesian prior distribution was set on the item parameters in order to aid convergence. A prior was used for all three quantitative analyses.

RESULTS

Verbal

Table 1 presents the results for the full information factor analysis of the verbal items. Both the second and third factors were clearly statistically significant at any commonly accepted level (beyond the .0001 level). It appears likely that additional factors might also have been statistically significant if a higher factor model had been run. It should be noted, however, that the percent of score variance explained for the three factors was 21.3, 3.9, and 3.2, respectively, indicating that the overwhelming plurality of the explained variance was determined by a dominant first factor.

Insert Table 1 About Here

A promax rotation of the three factor solution (Hendrickson & White, 1964) indicated that the factors were fairly highly correlated (r_{12} = .59; r_{13} = .65; and r_{23} = .59). The two factor solution with promax rotation yielded a .58 correlation between the factors.

Table 2 presents the factor loadings for the two and three factor solutions. If items were assigned to factors based on the factor on which they had the highest loading, then for the two factor solution, all reading comprehension and sentence correction items would have been assigned to the first factor. All but two of the analysis of situations items would have been assigned to the second factor. These two had slightly higher loadings on the first factor.

Insert Table 2 About Here

One way to test the interpretability of this solution is to compare the correlation between the promax rotated factors with the correlation between formula scores on the analysis of situations section and the combined sentence correction and reading comprehension sections. Since the factor scores contain some information from the items constituting the other factor and since the relative contribution of items that load heavily on the appropriate factor vary with the strength of their relationship to the factor, factor scores tend to be more reliable than formula-scores. This tends to increase the correlations between factors so that they are higher than the correlations between the observed scores from which those factors were derived. To the extent that the correlation between a pair of observed scores is close to the correlation between the corresponding pair of factor scores, then the item types constituting the observed scores define the factors. Since the factors correlate .58 and the combined sentence correction and reading comprehension section scores correlate .56 with analysis of situations scores, it is clear that it is item type that defines these two factors.

Assigning items to factors for the three-factor solution yields the same clearly defined first factor. All reading comprehension and sentence correction items loaded most heavily on the first factor. One analysis of situations item also loaded most heavily on the first factor, although its loadings on the second and third factor were similar in magnitude. The other analysis of situations items all loaded most heavily on either factor two (15 items) or factor three (19 items).

Five items had loadings of .7 or greater on factor two. Each of these items had the correct answer in the "C" position. Nine of the remaining ten items that loaded most heavily on factor two were keyed either "C" (4 items) or "E" (5 items). The remaining item had a key of "B", but its loading was low on all three factors, indicating that the score variance that was contributed was largely specific to that item and not to any common factors.

Six of the items had loadings of .7 or greater on factor three. Five of these items had an answer key of "A", and the last had a key of "D". All of the remaining 13 items that loaded most heavily on factor three were keyed "B" (6 items) or "D" (6 items).

Analysis of situations item share a common set of response options. "A" always indicates that the given item is a <u>major objective</u>. "B" indicates a <u>major factor</u>, "C" a <u>minor factor</u>, "D" a <u>major assumption</u>, and "E" an <u>unimportant issue</u>. Thus, it appears clear that the second and third factors are analysis of situations answer key-factors. Swinton and Powers (1981) found similar factors using classical exploratory factor analysis. Previous research on a similar item type called "analysis of explanations," which had previously been used in the Graduate Record Examinations Aptitude Test, indicates that such answer key factors are not uncommon (Kingston & Dorans, 1985; Swinton & Powers, 1980).

Quantitative

Table 3 presents the results of the full information factor analysis of the quantitative items. The first three factors are statistically significant at beyond the .0001 level. It is possible that additional significant factors might exist. The first factor is clearly dominant, explaining 33.7 percent of the observed score variance, compared to 3.5 percent for the second factor and 1.3 percent for the third factor.

Insert Table 3 About Here

Two-factor solution. A promox rotation of the two-factor solution showed that the factors correlated .72. Table 4 presents the factor loadings for the two- and three-factor solutions. For the two-factor solution, 37 of the 65 items loaded more heavily on the first factor and the remaining 28 items loaded more heavily on the second factor. Many items loaded about the same with each of the two factors. Some problem solving items loaded more heavily on the first factor and other loaded more heavily on the second. Similarly, different data sufficiency items loaded on each of the factors.

Insert Table 5 About Here

GMAT quantitative items can be fit into a three-way classification scheme: item type (problem solving or data sufficiency) mathematics type (arithmetic, algebra, or geometry) and problem type (pure or applied). Examination of the content of the quantitative items shed some light on the constructs or abilities underlying the factors. Table 5 presents the relationship between item content and factor loadings.

For the two-factor solution, all pure geometry items and most pure algebra items loaded more heavily on the first factor. Most applied algebra items, and applied and pure arithmetic items loaded more heavily on the second factor. The seven applied geometry items were split across the two-factors. Thus, the first factor might best be called a pure geometry and algebra factor, and the second factor might be called an arithmetic and applied algebra factor.

While the interpretation of the quantitative factors is not quite as clear as the interpretation of the verbal factors, this is not surprising. The first quantitative factor is relatively larger than the second quantitative factor compared to the corresponding relationship between verbal factors, and the correlation between the two quantitative factors is very high, about .72. In short, the two factor solution suggests that the bulk of explainable common variance in Quantitative scores for the GMAT population is due primarily to differences in the ability to perform geometric and algebraic manipulations in non-applied settings. Most of the remaining common variance cuts across quantitative item content, with the rest explained by variance due to the idiosyncratic item content or measurement error.

Three-factor solution. A promax rotation of the three factor solution showed that the factors were highly correlated (r_{12} = .61; r_{13} = .68; and r_{23} = .73). As with the two factor solution, there did not seem to be any consistent relationship between item type and factor loadings. Item content did appear to explain the factors that were found.

All pure geometry items and most pure algebra items had their largest loading the first-factor. Fifteen of the 16 applied arithmetic, problem solving items loaded most heavily on factor two but six of the eight applied arithmetic data sufficiency items had their largest loading on factor three. The other items were scattered among the three factors. Thus, the first factor can be characterized as pure geometry and pure algebra, the second factor as applied arithemtic, problem solving, and the third factor as applied arithmetic, data sufficiency. The latter two factors, however, were very weak, with the third factor explaining only about one percent of the score variance. Also, all three factors were fairly highly correlated.

In summary, both the two- and three-factor solutions supported the existance of a single dominant quantitative factor.

DISCUSSION AND CONCLUSION

For both the verbal items and quantitative items a dominant first factor and two additional, considerably weaker factors were found. This confirms the findings of Kingston, Leary, and Wightman (1985), but is a stronger analysis in that it is a direct nonlinear factor analytic approach as compared to the earlier approach based primarily on the comparision of item parameters estimated from heterogeneous and homogeneous subsets of items. Also, the use of full information factor analysis provided a statistical test of the factor model that confirmed the presence of multiple factors.

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Table 1
Full Information Factor Analysis of GMAT Verbal Items

Number Factors in Solution	Latent Root*	Percent Variance Explained*	χ^2	df	χ^2 Change**	df Change**	$P(\chi^2)$ Change***
1	24.0	21.3	406,006	4,829	---	---	---
2	3.7	3.9	402,533	4,745	3,473	84	.0000
3	2.6	3.2	399,794	4,662	2,734	83	.0000

*Latent roots and variance explained are from three factor solution

**Difference between χ^2 or degrees of freedom for this model and previous model

***Probability of change in χ^2 under null hypothesis

Table 2
Verbal Item Loadings

	Two-Factor solution		Three-Factor solution		
	1	2	1	2	3
RC 1	0.376	-0.049	0.411	-0.000	-0.070
RC 2	0.612	-0.012	0.614	0.033	-0.050
RC 3	0.255	0.007	0.270	0.002	-0.005
RC 4	0.696	-0.014	0.727	0.063	-0.111
RC 5	0.200	0.021	0.296	0.047	-0.030
RC 6	0.490	0.016	0.512	0.057	-0.057
RC 7	0.530	-0.065	0.520	-0.045	-0.007
RC 8	0.512	-0.094	0.531	-0.043	-0.076
RC 9	0.334	-0.023	0.343	0.016	-0.040
RC 10	0.436	-0.213	0.460	-0.160	-0.111
RC 11	0.430	-0.049	0.461	-0.000	-0.066
RC 12	0.447	0.030	0.466	0.053	-0.019
RC 13	0.402	-0.046	0.492	0.022	-0.000
RC 14	0.470	-0.023	0.498	0.003	-0.035
RC 15	0.256	-0.004	0.210	0.000	0.030
RC 16	0.143	-0.004	0.134	-0.074	0.033
RC 17	0.663	-0.092	0.651	-0.061	-0.019
RC 18	0.745	-0.129	0.716	-0.116	0.007
RC 19	0.664	-0.100	0.627	-0.070	0.005
RC 20	0.657	-0.049	0.655	-0.021	-0.013
RC 21	0.646	-0.091	0.600	-0.070	0.007
RC 22	0.665	-0.121	0.609	-0.101	-0.020
RC 23	0.660	-0.043	0.504	0.011	0.011
RC 24	0.614	-0.013	0.576	-0.099	0.100
RC 25	0.733	-0.152	0.603	-0.120	0.001
AO 1	0.070	0.235	0.025	-0.022	0.355
AO 2	0.267	0.307	0.207	0.000	0.342
AO 3	0.230	0.371	0.264	0.314	0.003
AO 4	0.027	0.349	-0.023	0.103	0.363
AO 5	0.112	0.439	0.095	0.231	0.299
AO 6	0.031	0.371	-0.025	0.092	0.407
AO 7	0.249	0.374	0.046	-0.166	0.025
AO 8	-0.211	0.737	-0.070	0.774	-0.061
AO 9	0.331	0.479	0.064	-0.067	0.065
AO 10	-0.201	0.706	-0.090	0.714	-0.011
AO 11	0.199	0.399	0.204	0.269	0.100
AO 12	0.356	0.403	0.261	0.100	0.534
AO 13	-0.006	0.502	0.007	0.560	-0.099
AO 14	-0.231	0.726	-0.069	0.053	-0.206
AO 15	0.075	0.451	0.050	0.230	0.310
AO 16	-0.273	0.052	-0.112	0.926	-0.133
AO 17	0.060	0.097	0.110	0.172	-0.097
AO 18	0.099	0.415	-0.056	-0.009	0.769
AO 19	0.220	0.309	0.200	0.221	0.250
AO 20	0.126	0.097	0.105	0.059	0.005
AO 21	0.149	0.205	0.044	-0.060	0.533
AO 22	0.100	0.320	0.103	0.173	0.214
AO 23	-0.032	0.500	0.033	0.505	0.009
AO 24	0.111	0.303	0.130	0.251	0.002
AO 25	0.064	0.160	0.000	0.104	0.030
AO 26	-0.304	0.923	-0.159	1.023	-0.192
AO 27	0.052	0.321	0.002	0.273	0.076
AO 28	0.092	0.400	-0.073	-0.125	0.017
AO 29	0.234	0.432	0.191	0.175	0.382
AO 30	0.019	0.010	0.086	0.417	0.151
AO 31	0.077	0.467	-0.107	-0.095	0.002
AO 32	0.371	0.346	0.215	-0.057	0.635
AO 33	0.166	0.332	0.088	0.020	0.453
AO 34	-0.109	0.625	-0.037	0.500	0.076
AO 35	0.199	0.206	0.000	0.266	0.045
SC 1	0.305	-0.049	0.380	-0.032	-0.021
SC 2	0.240	-0.004	0.245	-0.040	-0.014
SC 3	0.351	-0.059	0.372	0.005	-0.004
SC 4	0.309	0.094	0.330	0.077	0.022
SC 5	0.385	0.010	0.399	0.054	-0.032
SC 6	0.410	-0.047	0.398	-0.019	-0.029
SC 7	0.440	-0.008	0.45[illegible]	0.026	-0.039
SC 8	0.250	0.044	0.262	0.022	0.020
SC 9	9.474	-0.046	0.472	-0.026	-0.032
SC 10	0.440	0.023	0.460	0.033	0.003
SC 11	0.434	0.070	0.408	0.086	0.038
SC 12	0.376	-0.007	0.306	-0.001	0.010
SC 13	0.533	-0.075	0.531	-0.076	0.002
SC 14	0.293	0.109	0.279	0.059	0.090
SC 15	0.262	0.100	0.279	0.114	-0.005
SC 16	0.363	0.003	0.374	0.010	-0.019
SC 17	0.434	0.018	0.438	0.019	0.001
SC 18	0.428	0.053	0.419	0.040	0.024
SC 19	0.522	0.019	0.523	-0.029	0.070
SC 20	0.662	-0.016	0.638	-0.095	0.111
SC 21	0.521	-0.066	0.470	-0.036	-0.018
SC 22	0.564	-0.043	0.500	-0.072	0.010
SC 23	0.600	-0.050	0.664	0.007	-0.046
SC 24	0.457	0.033	0.424	-0.023	0.104
SC 25	0.548	-0.098	0.509	-0.111	0.084

Table 3
Full Information Factor Analysis of GMAT Quantitative Items

Number Factors in Solution	Latent Root*	Percent Variance Explained*	χ^2	df	χ^2 Change**	df Change**	$P(\chi^2)$ Chang
1	26.9	33.7	245,107	4,868	---	---	---
2	2.0	3.5	244,006	4,804	1,101	64	.0000
3	1.4	1.3	243,339	4,741	668	63	.0000

*Latent roots and variance explained are from three factor solution

**Difference between χ^2 or degrees of freedom for this model and previous model

***Probability of change in χ^2 under null hypothesis

Table 4
Quantitative Item Loadings
Two-Factor Solution

Items*	Content Classification	Two-Factor solution 1	2	Three-Factor Solution 1	2	3
P1 1	Algebra - Pure	0.249	0.287	0.244	-0.104	0.434
P1 2	Arithmetic - Pure	0.145	0.381	0.143	0.288	0.149
P1 3	Geometry - Pure	0.269	0.231	0.278	0.011	0.236
P1 4	Algebra - Applied	0.004	0.465	0.008	0.265	0.241
P1 5	Arithmetic - Applied	0.166	0.327	0.137	0.228	0.184
P1 6	Arithmetic - Applied	0.049	0.459	0.019	0.331	0.228
P1 7	Arithmetic - Applied	-0.273	0.767	-0.278	0.499	0.341
P1 8	Algebra - Applied	-0.065	0.733	-0.129	0.435	0.450
P1 9	Arithmetic - Applied	-0.051	0.409	-0.045	0.442	0.023
P1 10	Arithmetic - Applied	-0.046	0.537	-0.044	0.285	0.304
P1 11	Arithmetic - Applied	0.115	0.561	0.087	0.479	0.181
P1 12	Arithmetic - Pure	0.274	0.230	0.260	0.088	0.152
P1 13	Arithmetic - Applied	-0.106	0.439	-0.115	0.411	0.090
P1 14	Algebra - Pure	0.474	0.222	0.438	0.165	0.141
P1 15	Arithmetic - Pure	0.360	0.384	0.308	0.259	0.248
P1 16	Geometry - Applied	0.671	0.207	0.631	0.289	0.014
P1 17	Algebra - Pure	0.311	0.104	0.289	0.220	-0.063
P1 18	Arithmetic - Applied	0.402	0.446	0.297	0.550	0.046
P1 19	Geometry - Pure	0.728	0.082	0.683	0.218	-0.046
P1 20	Geometry - Applied	0.449	0.377	0.430	0.487	-0.055
P2 1	Arithmetic - Applied	-0.147	0.469	-0.146	0.419	0.109
P2 2	Arithmetic - Applied	0.213	0.393	0.200	0.270	0.186
P2 3	Algebra - Pure	0.066	0.256	0.060	-0.007	0.297
P2 4	Arithmetic - Pure	0.547	0.063	0.551	-0.192	0.258
P2 5	Geometry - Pure	0.633	-0.006	0.630	-0.080	0.092
P2 6	Geometry - Applied	0.292	0.341	0.304	0.238	0.136
P2 7	Arithmetic - Applied	0.185	0.444	0.179	0.504	0.006
P2 8	Arithmetic - Pure	-0.062	0.500	-0.100	0.221	0.378
P2 9	Arithmetic - Pure	0.626	-0.024	0.624	-0.124	0.114
P2 10	Algebra - Applied	0.005	0.504	0.029	0.453	0.083
P2 11	Arithmetic - Applied	0.249	0.510	0.234	0.516	0.071
P2 12	Arithmetic - Applied	0.097	0.488	0.047	0.469	0.141
P2 13	Arithmetic - Applied	0.379	0.378	0.369	0.411	0.028
P2 14	Arithmetic/Algebra-Applied	0.585	0.310	0.531	0.364	0.066
P2 15	Algebra - Pure	0.457	0.451	0.451	0.335	0.196
P2 16	Geometry - Applied	0.288	0.424	0.224	0.524	0.012
P2 17	Geometry - Pure	0.884	-0.019	0.891	0.181	-0.169
P2 18	Algebra - Applied	0.632	0.120	0.626	0.214	-0.051
P2 19	Algebra - Pure	0.949	-0.148	0.922	-0.055	-0.038
P2 20	Arithmetic - Applied	0.396	0.453	0.347	0.601	-0.071
DS 1	Arithmetic - Applied	-0.380	0.740	-0.406	0.210	0.599
DS 2	Algebra - Pure	0.136	0.272	0.130	0.085	0.217
DS 3	Arithmetic - Pure	-0.034	0.58[illegible]	-0.080	-0.023	0.687
DS 4	Arithmetic - Applied	-0.067	0.515	-0.104	0.097	0.492
DS 5	Geometry - Applied	0.235	0.351	0.193	0.081	0.384
DS 6	Algebra - Applied	-0.172	0.529	-0.220	-0.024	0.641
DS 7	Algebra - Applied	0.134	0.397	0.135	0.341	0.114
DS 8	Geometry - Applied	0.361	0.142	0.348	0.016	0.169
DS 9	Arithmetic - Applied	-0.074	0.360	0.116	0.026	0.404
DS 10	Arithmetic/Geometry-Applied	0.362	0.313	0.293	-0.004	0.431
DS 11	Arithmetic - Applied	-0.056	0.370	-0.089	0.141	0.297
DS 12	Geometry - Pure	0.478	0.234	0.426	0.053	0.283
DS 13	Arithmetic - Applied	0.048	0.300	-0.004	0.042	0.349
DS 14	Algebra - Applied	-0.131	0.524	-0.139	0.463	0.133
DS 15	Geometry - Pure	0.279	0.195	0.277	0.176	0.056
DS 16	Arithmetic - Applied	0.278	0.276	0.249	0.464	-0.120
DS 17	Algebra - Pure	0.439	0.072	0.433	0.035	0.072
DS 18	Algebra - Pure	0.867	-0.164	0.802	0.020	-0.073
DS 19	Arithmetic - Pure	0.096	0.538	0.033	0.393	0.281
DS 20	Algebra - Pure	0.276	0.195	0.160	-0.094	0.443
DS 21	Geometry - Applied	0.502	0.004	0.509	0.150	-0.147
DS 22	Geometry - Pure	0.599	0.184	0.547	0.290	-0.020
DS 23	Algebra - Pure	0.435	0.310	0.410	0.273	0.097
DS 24	Arithmetic - Applied	0.435	0.369	0.369	0.407	0.070
DS 25	Geometry - Pure	0.843	-0.022	0.783	0.200	-0.125

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*P1 - Problem Solving, First section
P2 - Problem solving, second section
DS - Data Sufficiency

Table 5
Quantitative Measure
Content Breakdown

			Two-Factor Solution Factor with Larger Loading		Three-Factor Solution Factor with Largest Loading		
Content	Situation	Item-Type	1	2	1	2	3
Algebra	Pure	Problem Solving	4	1	4	-	1
Albegra	Pure	Data Dufficiency	4	1	3	-	2
Algebra	Applied	Problem Solving	1	3	1	2	1
Algebra	Applied	Data Sufficiency	-	3	-	2	1
Arithmetic	Pure	Problem Solving	2	4	3	1	2
Arithmetic	Pure	Data Sufficiency	-	2	-	1	2
Arithmetic	Applied	Problem Solving	2	14	1	15	-
Arithmetic	Applied	Data Sufficiency	3	5	0	2	6
Geometry	Pure	Problem Solving	4	-	4	-	-
Geometry	Pure	Data Sufficiency	4	-	4	-	-
Geometry	Applied	Problem Solving	2	2	2	2	-
Geometry	Applied	Data Sufficiency	2	1	2	-	1